Summary Statistics

Exploring Data

June 15th, 2021

Where are we?

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So far we've looked at how we can represent and manipulate data.

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What are we doing today?

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1. We want *descriptive statistics*

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2. For a variable, what does this mean?

What are we doing today?

1. We want *descriptive statistics*
2. For a variable, what does this mean?
3. What are *correlation* of two variables?

Description for a variable

We have two main descriptors:

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1. Location: mean, median, mode

Description for a variable

We have two main descriptors:

1. Location: mean, median, mode
2. Dispertion: variance, standard deviation

MEASURES OF LOCATION

These are 30 hours of average defect data on sets of circuit boards. Roughly what is the typical value?

1.45	1.65	1.50	2.25	1.65	1.60	2.30	2.20	2.70	1.70
2.35	1.70	1.90	1.45	1.40	2.60	2.05	1.70	1.05	2.35
1.90	1.55	1.95	1.60	2.05	2.05	1.70	2.30	1.30	2.35

Location and central tendency

- There exists a distribution of values
- We are interested in the “center” of the distribution

Two measures are the sample mean and the sample median

They look similar, and measure the same thing

They differ systematically (and predictably) when the data are not symmetric

[WG]

THE MEAN OF AGGREGATE DATA

State	Listing	IncomePC	State	Listing	IncomePC	State	Listing	IncomePC
Hawaii	896800	24057	Rhode Island	432534	22251	Texas	266388	19857
California	713864	22493	Delaware	420845	22828	Mississippi	255774	15838
New York	668578	25999	Oregon	417551	20419	Tennessee	255064	19482
Connecticut	654859	29402	Idaho	415885	18231	Wisconsin	243006	21019
Dist.Columbia	577921	31136	Illinois	377683	23784	Michigan	241107	22333
Nevada	549187	24023	New Hampshire	361691	23434	Missouri	221773	20717
New Jersey	529201	23038	New Mexico	358369	17106	South Dakota	220708	19577
Massachusetts	521769	25616	Vermont	346469	20224	West Virginia	219275	17208
Wyoming	499674	20436	South Carolina	340066	17695	Arkansas	217659	16898
Maryland	480578	24933	North Carolina	330432	19669	Ohio	209189	20928
Utah	475060	17043	Georgia	326699	20251	Kentucky	208391	17807
Colorado	467979	22333	Alaska	324774	23788	Oklahoma	203926	17744
Arizona	448791	19001	Minnesota	306009	22453	Kansas	201389	20896
Florida	447698	21677	Maine	299796	19663	Indiana	200683	20378
Montana	446584	17865	Pennsylvania	295133	22324	Iowa	184999	20265
Virginia	443618	22594	Louisiana	280631	17651	North Dakota	173977	18546
Washington	440542	22610	Alabama	269135	18010	Nebraska	164326	20488

Average list price:
1/51 ($898,800 + $713,864 + … + $164,326) = $369,687

[WG]

AVERAGING AVERAGES?

Hawaii's average listing = $896,800

Hawaii's population = 1,275,194

Illinois' average listing = $377,683

Illinois' population = 12,763,371

Illinois and Hawaii each get an equal weight of 1/51 = .019607 when the mean is computed.

Looks like Hawaii is getting too much influence …





[WG]

WEIGHTED AVERAGE

$$\text{Simple average} = \overline{\text{Listing}} = \sum_{\text{States}} \text{Weight}_{\text{State}}\ \text{Listing}_{\text{State}}$$

$$\text{Weight} = \frac{1}{51} = .019607$$

Illinois is 10 times as big as Hawaii. Suppose we use weights that are in proportion to the state's population. (The weights sum to 1.0.)

$\text{Weight}_{\text{State}}$ varies from .001717 for Wyoming to .121899 for California

New average is \$409,234 compared to \$369,687 without weights, an error of 11%

Sometimes an unequal weighting of the observations is necessary

State population data: http://www.factmonster.com/ipka/A0004986.html

AVERAGES & TIME SERIES

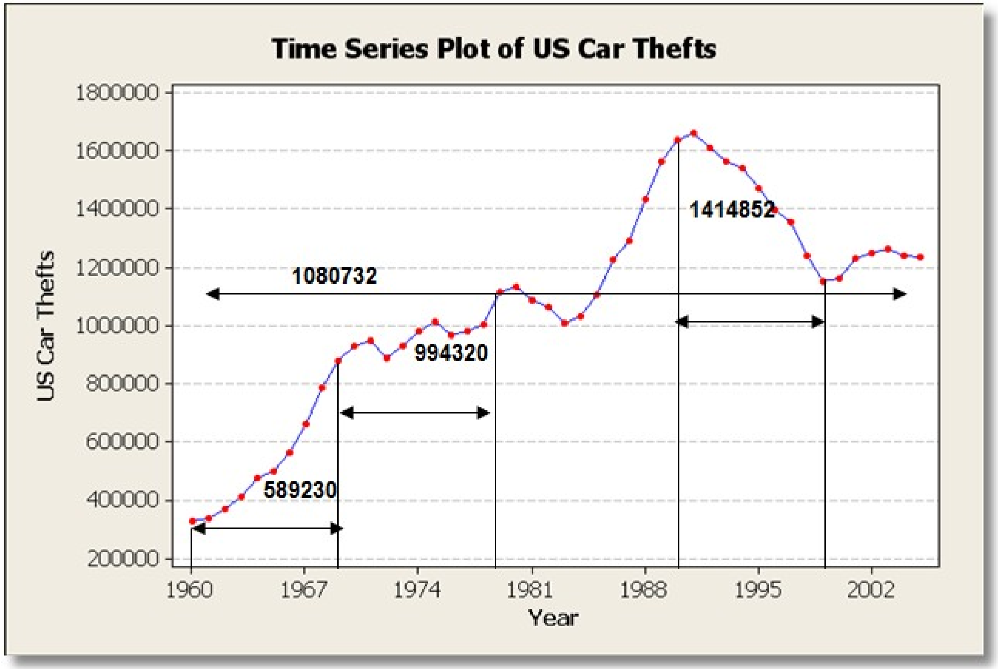
Averaging trending time series is usually not helpful

Mean changes completely depending on time interval

What about periodic time series data ??????????

Ask yourself:

- **Does the mean over the entire observation period mean anything?**
- **Does it estimate anything meaningful?**



THE SAMPLE MEDIAN

Median:

- Sort the data
- Take the middle point*

Odd number:

- Central observation: Med[1,2,4,6,8,9,17]

Even number:

- Midpoint between the two central observations
  Med[1,2,4,6,8,9,14,17] = (6+8)/2=7

* CMSC351 will show you how to find the median in linear time!

WHAT IS THE CENTER?

The mean and median measure the central tendency of data

Generally, the center of of a dataset is a point in its range that is close to the data.

Close? Need a distance metric between two points x and x_2

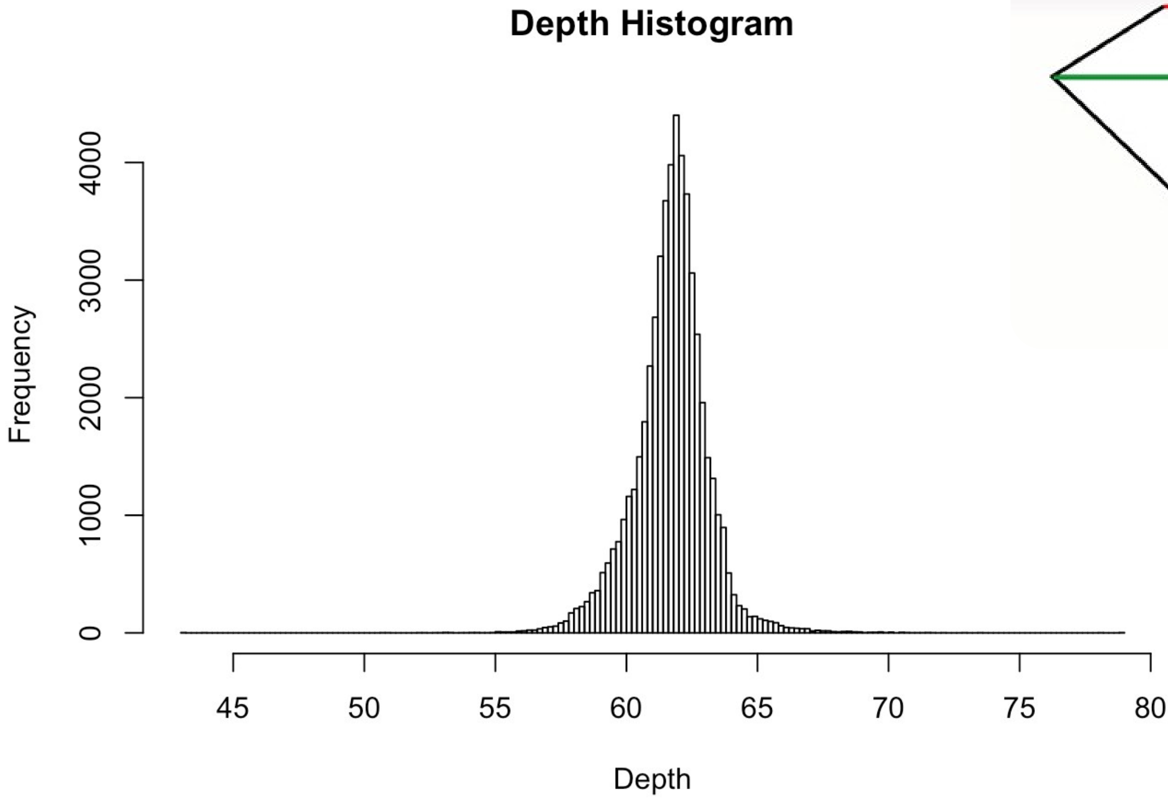
We've talked about some already!

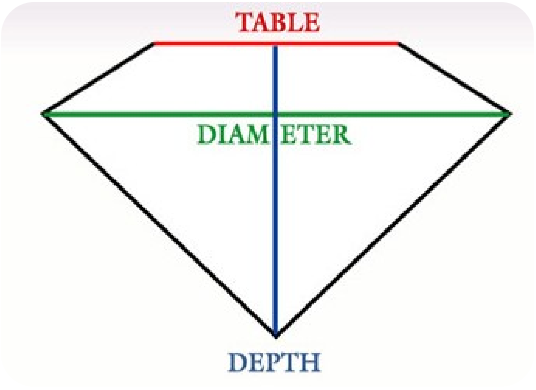
- Absolute deviation: $| x_1 - x_2 |$
- Squared deviation: $(x_1 - x_2)^2$

We'll define the center based on these metrics

DATASET FOR THIS PART

53,940 measurements of diamonds





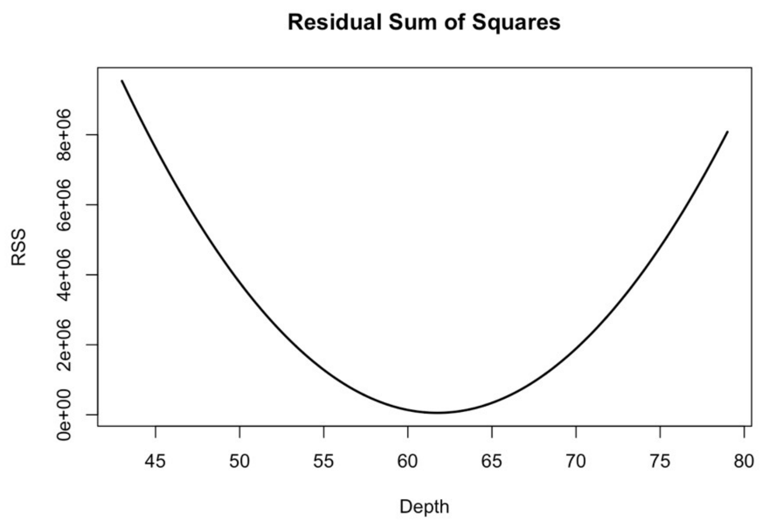
More info: https://en.wikipedia.org/wiki/Diamond_(gemstone)#Gemological_characteristics

THE MEAN REVISITED

Define a center point μ based on some function of the distance from each data point to that center point

- Residual sum of squares (RSS) for a point μ:

$$RSS(\mu) = \frac{1}{2}\sum_{i=1}^{n}(x_i - \mu)^2$$



So what should our estimate of the "center" of this dataset be, based on the RSS metric? ?????????????

THE MEAN REVISITED

Want the point μ that minimizes the RSS ??????????

- Find the derivative of RSS and set it to zero, solve for μ!

$$\frac{\partial}{\partial \mu} \frac{1}{2} \sum_{i=1}^{n} (x_i - \mu)^2 = \frac{1}{2} \sum_{i=1}^{n} \frac{\partial}{\partial \mu} (x_i - \mu)^2$$

$$= \frac{1}{2} \sum_{i=1}^{n} 2(x_i - \mu) \times (-1)$$

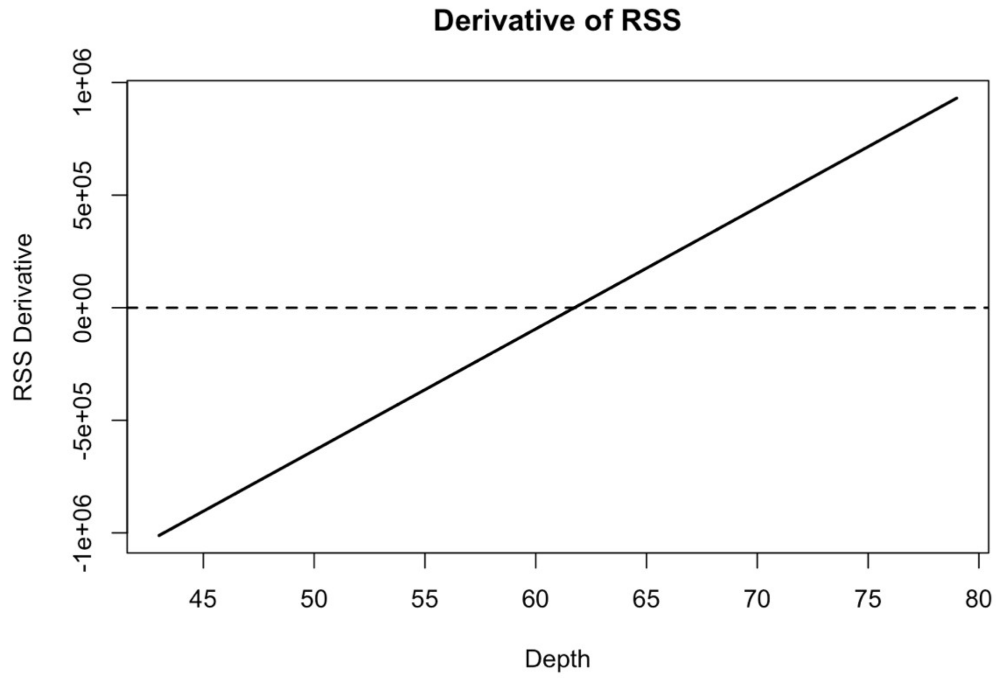
THE MEAN REVISITED

$$= \frac{1}{2}\sum_{i=1}^{n} 2(x_i - \mu) \times (-1)$$

$$= \frac{1}{2} 2 \sum_{i=1}^{n} (\mu - x_i)$$

$$= \sum_{i=1}^{n} \mu - \sum_{i=1}^{n} x_i$$

$$= n\mu - \sum_{i=1}^{n} x_i$$



THE MEAN REVISITED

Set the derivative to zero and solve for μ:

$$\frac{\partial}{\partial \mu} = 0$$

$$n\mu - \sum_{i=1}^{n} x_i = 0$$

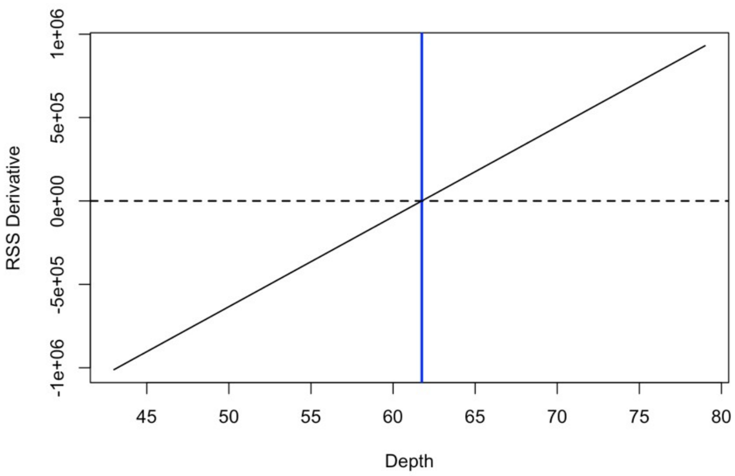
$$n\mu = \sum_{i=1}^{n} x_i$$

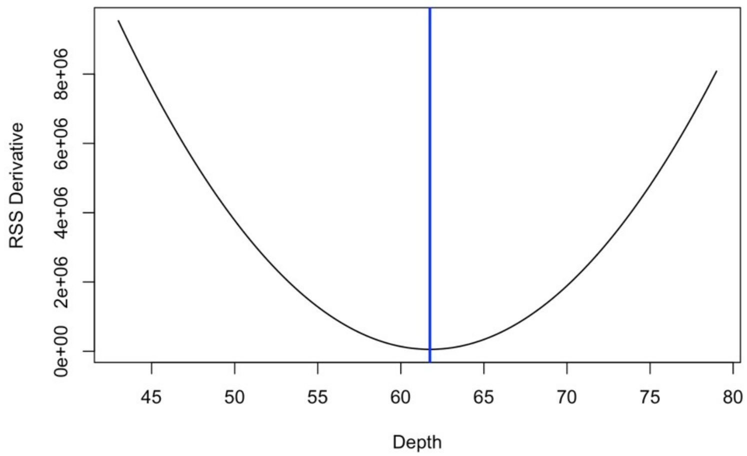
$$\mu = \frac{1}{n} \sum_{i=1}^{n} x_i$$

The mean is the point μ that minimizes the RSS for a dataset.

THE MEAN REVISITED

What about a weighted average ???????





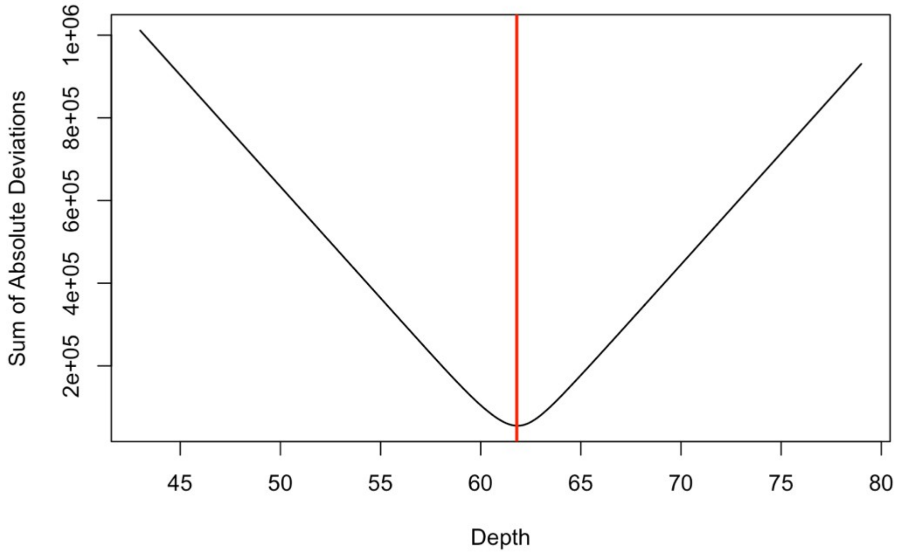
The mean is the point μ that minimizes the RSS for a dataset.

THE MEDIAN REVISITED

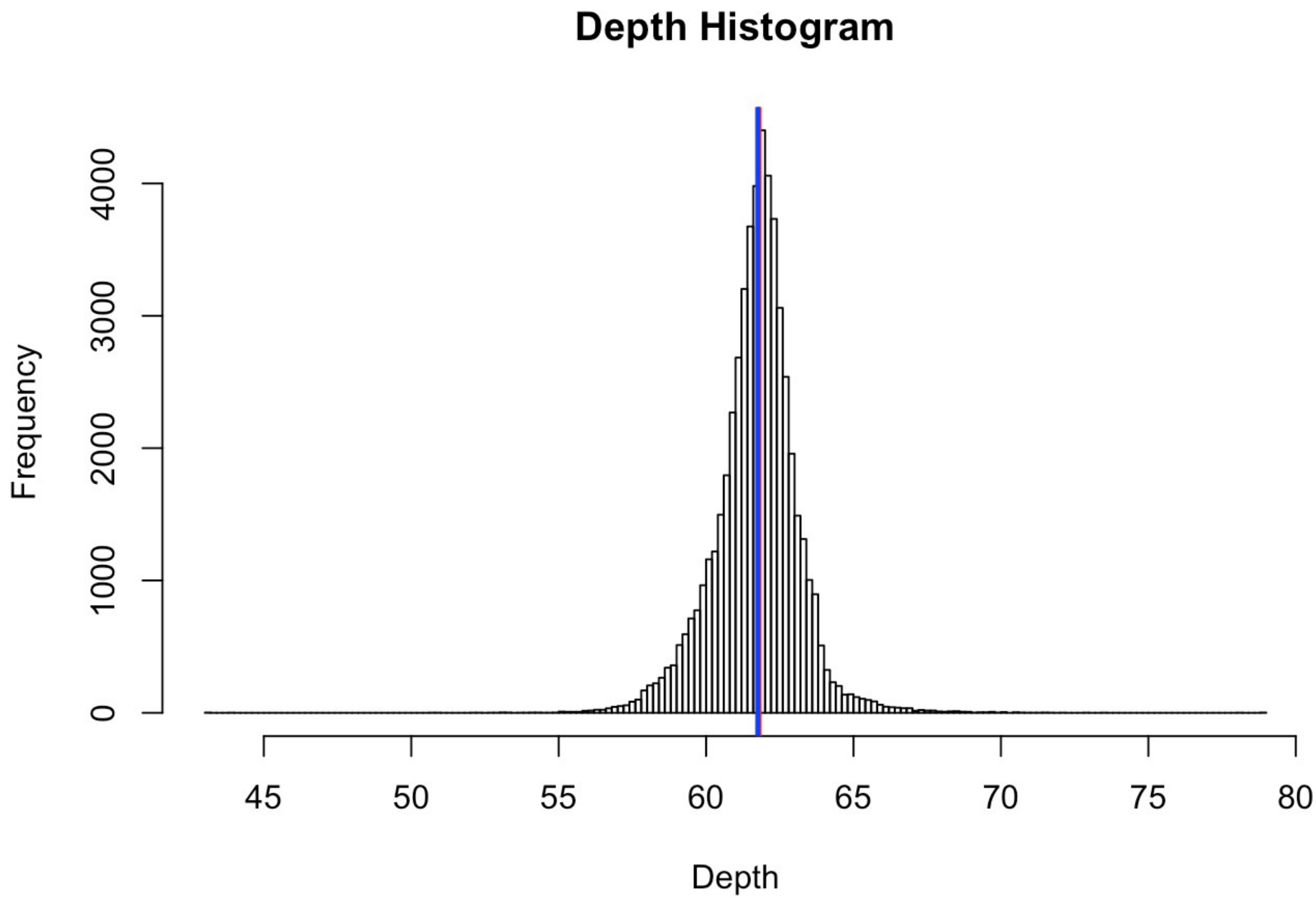
Define a center point *m* based on some function of the distance from each data point to that center point

- **The median *m* minimizes the sum of absolute differences:**

$$\sum_{i=1}^{n} |x_i - m|$$



MEAN != MEDIAN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Frequency
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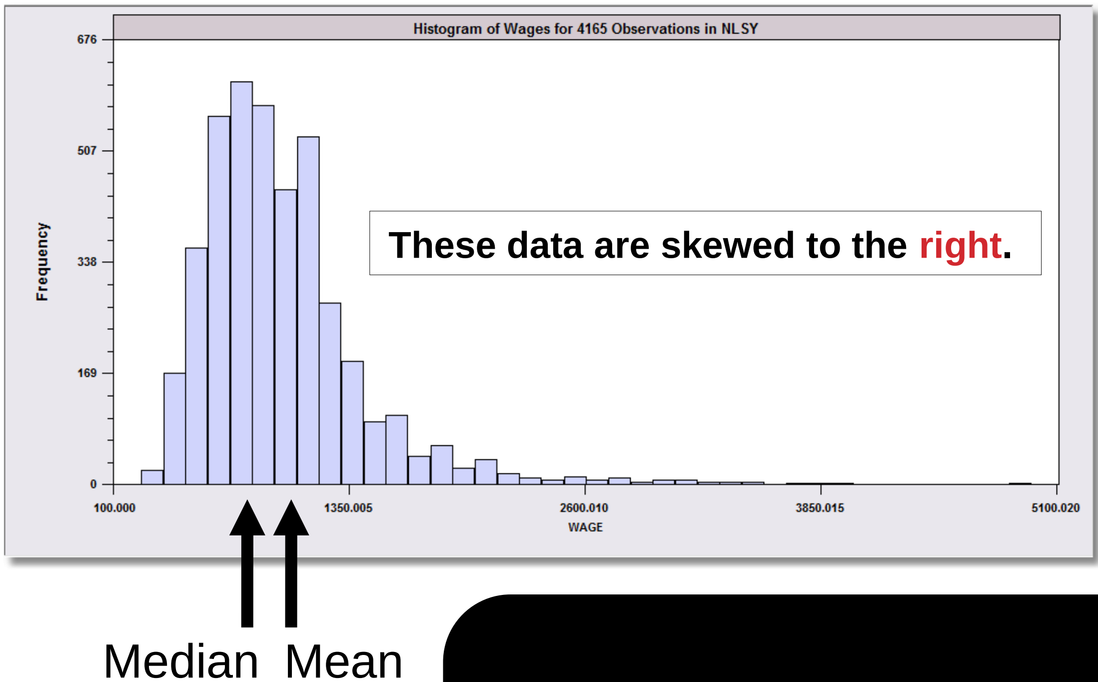
SKEWED DATA



Monthly Earnings
N = 595,
Median = 800
Mean = 883

The mean will exceed the median when the distribution is skewed to the right.

Skewness is in the direction of the long tail

SKEWNESS

Extreme observations distort means but not medians.

Outlying observations distort the mean:

- Med [1,2,4,6,8,9,17] = 6
- Mean[1,2,4,6,8,9,17] = 6.714
- Med [1,2,4,6,8,9,17000] = 6 (still)
- Mean[1,2,4,6,8,9,17000] = 2432.8 (!)

Typically occurs when there are some outlying observations, such as in cross sections of income or wealth and/or when the sample is not very large.

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The New York Times

Business Day

DATAPOINTS

Income Gap Grows Wider (and Faster)

By ANNA BERNASEK
Published: August 31, 2013

INCOME inequality in the United States has been growing for decades, but the trend appears to have accelerated during the Obama administration. One measure of this is the relationship between median and average wages.

1.7%

Increase in **median** annual wage

3.9%

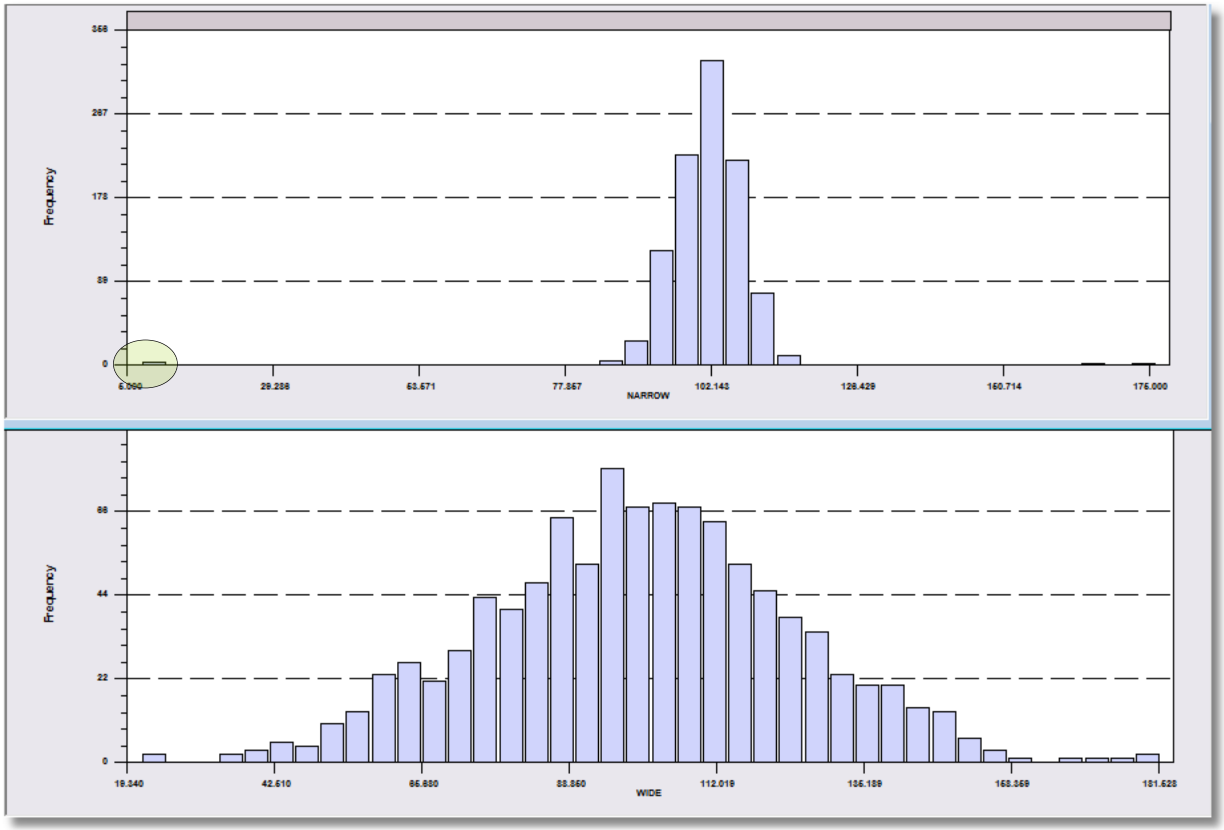
Increase in **average** annual wage

2009 through 2011

The median wage is straightforward: it's the midpoint of everyone's wages. Interpreting the average, though, can be tricky. If the income of a handful of people soars while everyone else's remains the same, the entire group's average may still rise substantially. So when average wages grow faster than the median, as happened from 2009 through 2011, it means that lower earners are falling further behind those at the top.

One way to see the acceleration in inequality is to look at the ratio of average to median annual wages. From 2001 through 2008, during the George W. Bush administration, that ratio grew at 0.28 percentage point per year. From 2009 through 2011, the latest year for which the data is available, the ratio increased 1.14 percentage points annually, or roughly four times faster.

MORE INFORMATION NEEDED!

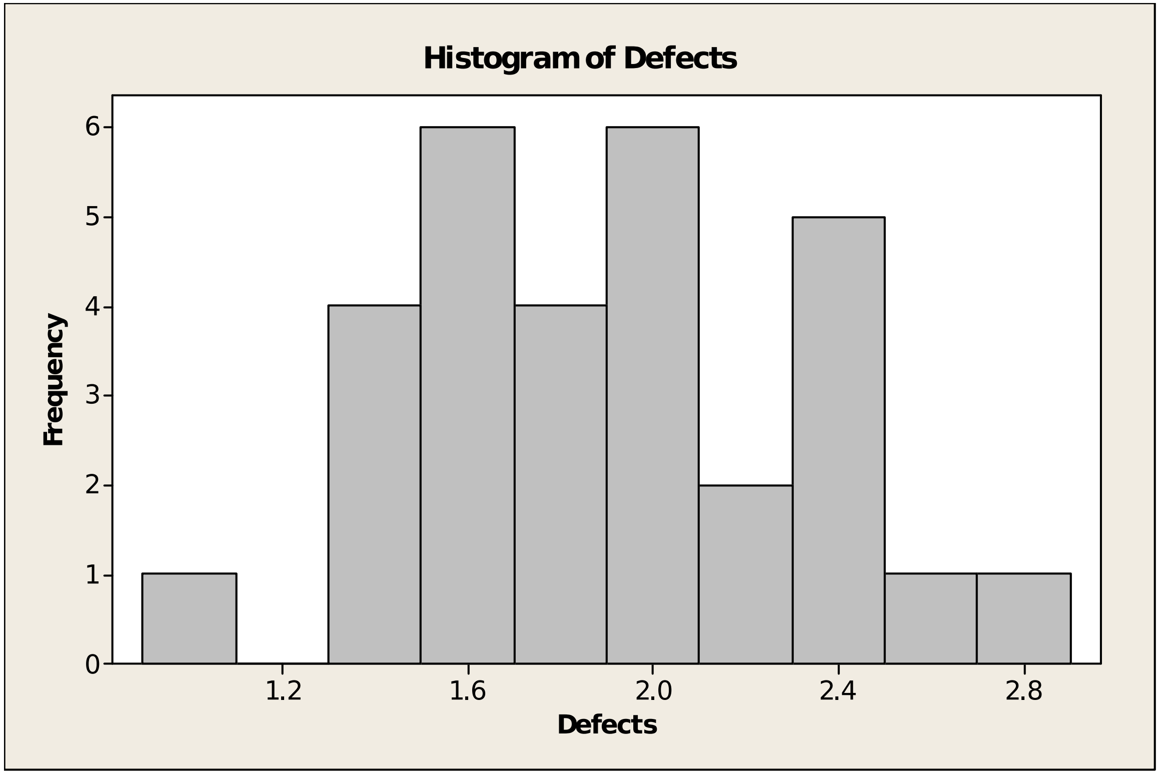


Both data sets have a mean of about 100.

DISPERSION OF THE OBSERVATIONS

30 hours of average defect data on sets of circuit boards.

1.45	1.65	1.50	2.25	1.65	1.60	2.30	2.20	2.70	1.70
2.35	1.70	1.90	1.45	1.40	2.60	2.05	1.70	1.05	2.35
1.90	1.55	1.95	1.60	2.05	2.05	1.70	2.30	1.30	2.35



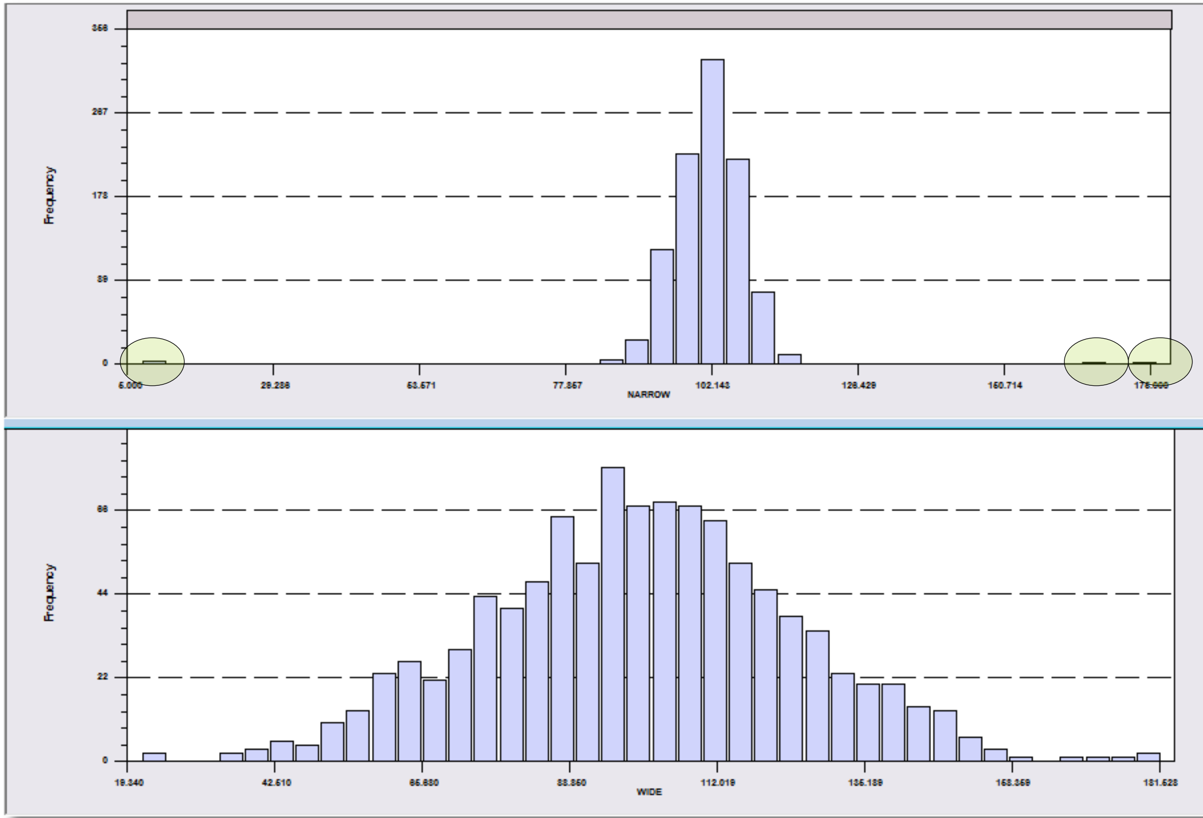
We quantify the variation of the values around the mean.

Note the range is from 1.05 to 2.70. This gives an idea where the data lie.

The mean plus a measure of the variation do the same job.

RANGE AS A MEASURE OF DISPERSION

Problems ????????



These two data sets both have 1,000 observations that range from about 10 to about 180.

VARIANCE & STDEV: UNIVARIATE MEASURES OF DISPERSION

Variance = s_x^2 = $\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2$ or $\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \bar{x})^2$

Standard deviation = s_x = $\sqrt{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})^2}$

The variance is commonly used statistic for spread

- What are the units of the variance ??????????

Standard deviation "fixes this," can be used as an interpretable unit of measurement

Why n-1? http://nebula.deanza.edu/~bloom/math10/m10divideby_nminus1.pdf

VARIANCE, ASIDE: WHY DIVIDE BY N-1?

Remember: we are typically calculating the mean / median / variance / etc of a sample of a population

- Want that {mean, median, variance, …} to be an “unbiased” estimate of the true population’s {mean, median, variance, …}

Unbiased? Consider variance …

1. Look at every possible sample of the population
2. Compute sample variance of each population
3. Is the average of those variances equal to the population variance? If so, then this is an “unbiased” estimator.

VARIANCE, ASIDE: WHY DIVIDE BY N-1?

Dividing by n-1 in the sample variance computation leads to an unbiased estimate of the population variance

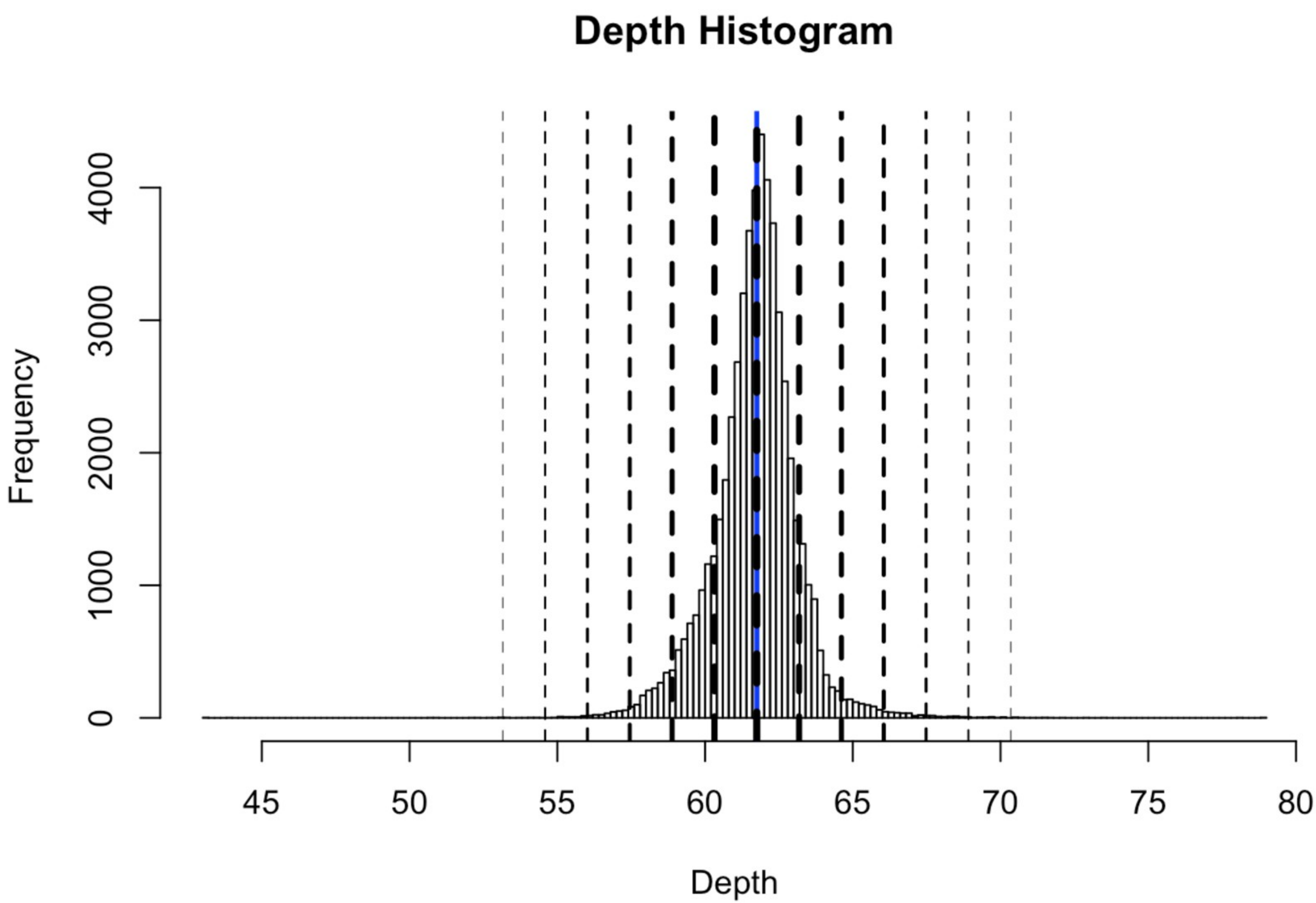
$$\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

Intuition. Fix a sample …

- Variance measures distribution around a mean
- Sampled values are, on average, closer to sample mean than to true population mean
- So, we will underestimate the true variance slightly
- Using n-1 instead of n makes our variance calculation bigger

This "embiggening" impacts smaller *n* more than larger *n*

- Larger samples are better estimates of population
- If sample is the population, just divide by *n* …


Depth Histogram
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0
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80
Depth

USING "STANDARD DEVIATIONS FROM THE MEAN" AS A UNIT

SDs	Proportion	Interpretation
1	0.68	68% of the data is within ± 1 sds
2	0.95	95% of the data is within ± 2 sds
3	0.9973	99.73% of the data is within ± 3 sds
4	0.999937	99.9937% of the data is within ± 4 sds
5	0.9999994	99.999943% of the data is within ± 5 sds
6	1	99.9999998% of the data is within ± 6 sds

PAIRS OF DATA POINTS?

CORRELATION

Variables Y and X vary together

Causality vs. correlation: Does movement in X "cause" movement in Y in some metaphysical sense?

Correlation

- Simultaneous movement through a statistical relationship
- Simultaneous variation "induced" by the variation of a common third effect

HOUSE PRICES & PER CAPITA INCOME

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Connecticut	654859	29402
Dist.Columbia	577921	31136
Nevada	549187	24023
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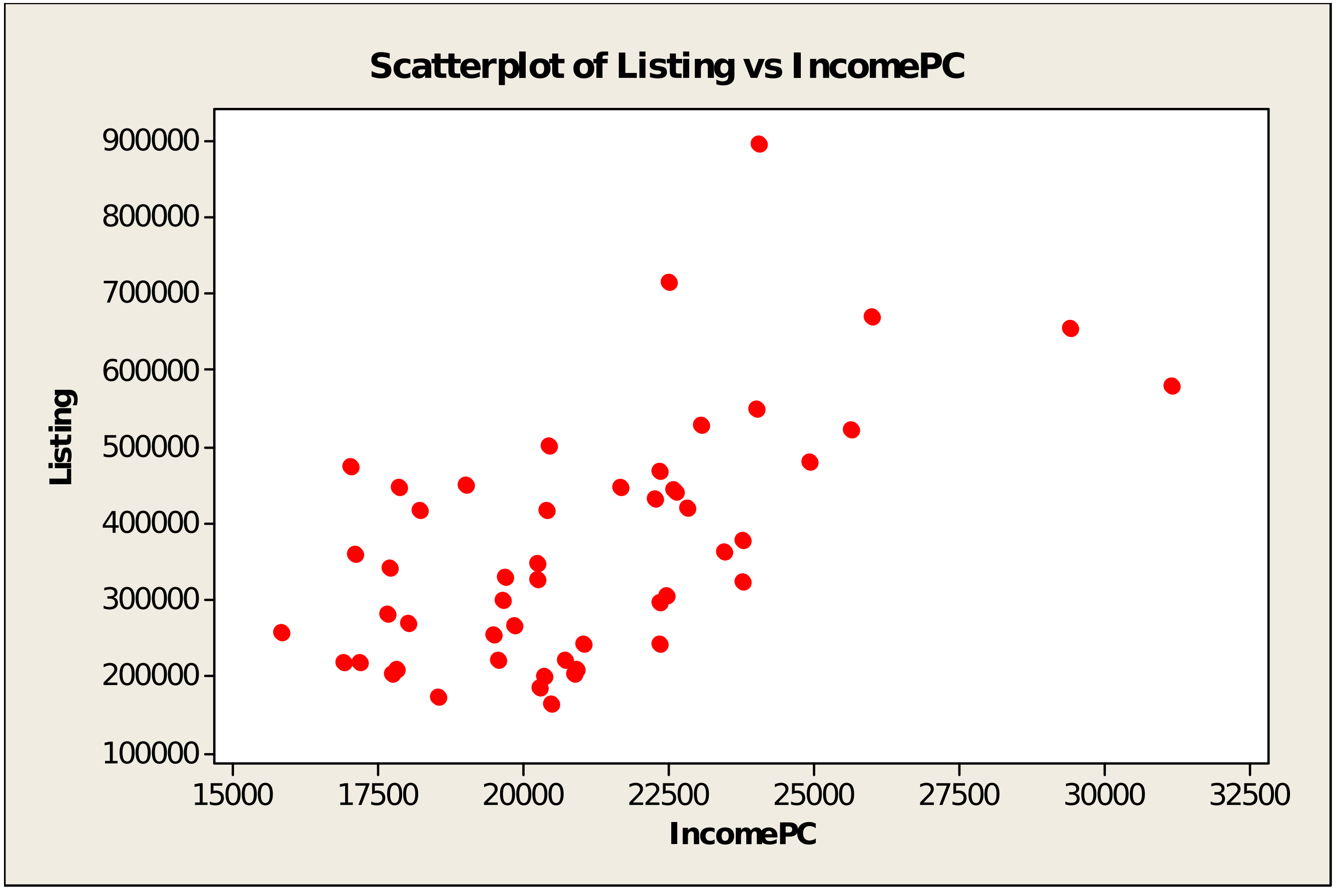
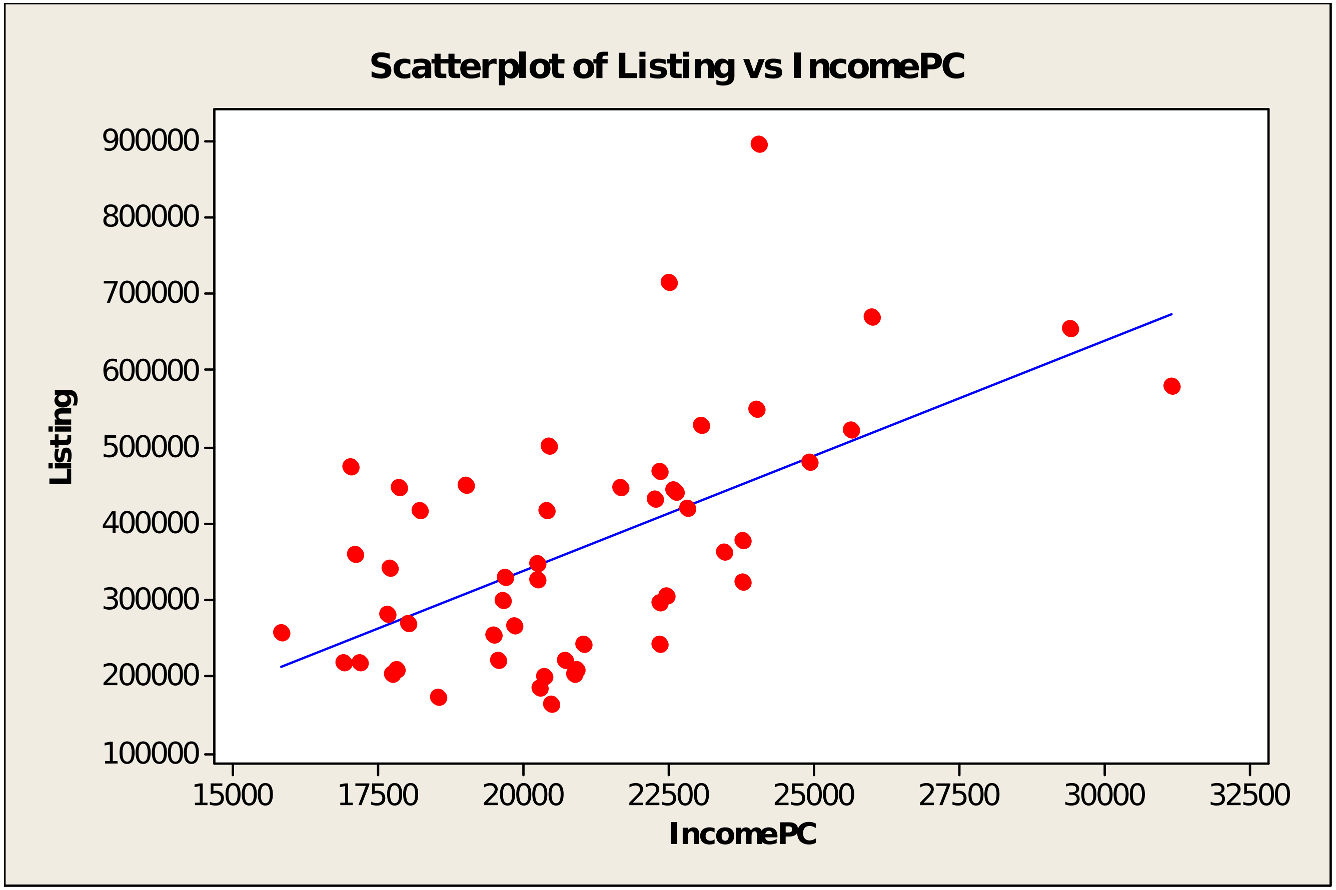
Indiana	200683	20378
Iowa	184999	20265
North Dakota	173977	18546
Nebraska	164326	20488

SCATTER PLOT SUGGESTS POSITIVE CORRELATION

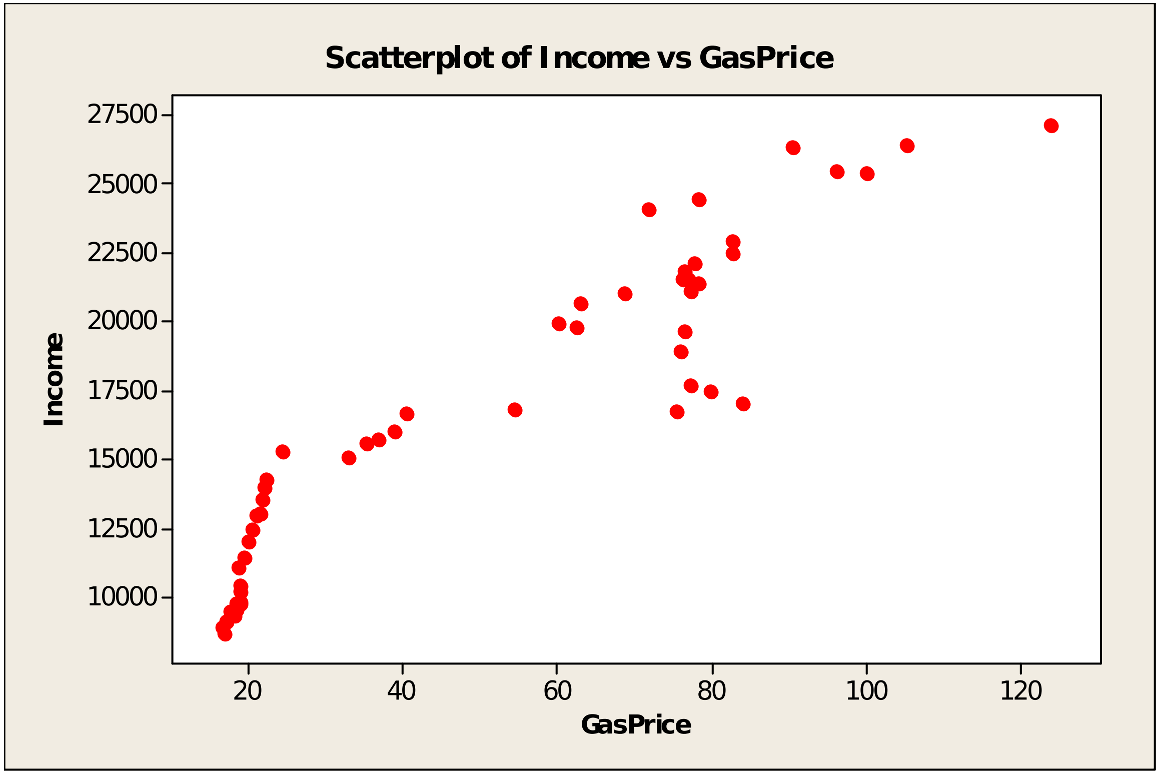


LINEAR REGRESSION MEASURES CORRELATION



CORRELATION IS NOT CAUSATION

Price and income seem to be positively correlated.

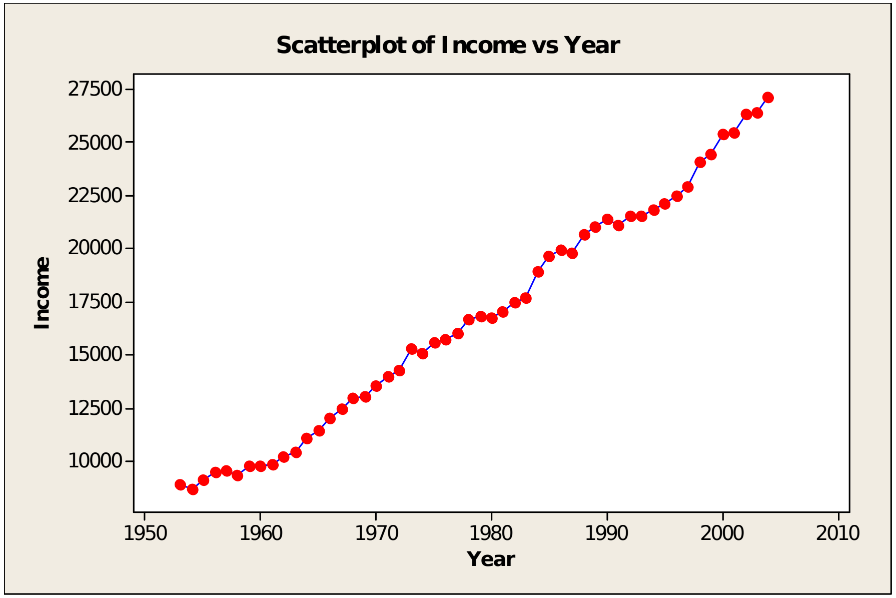


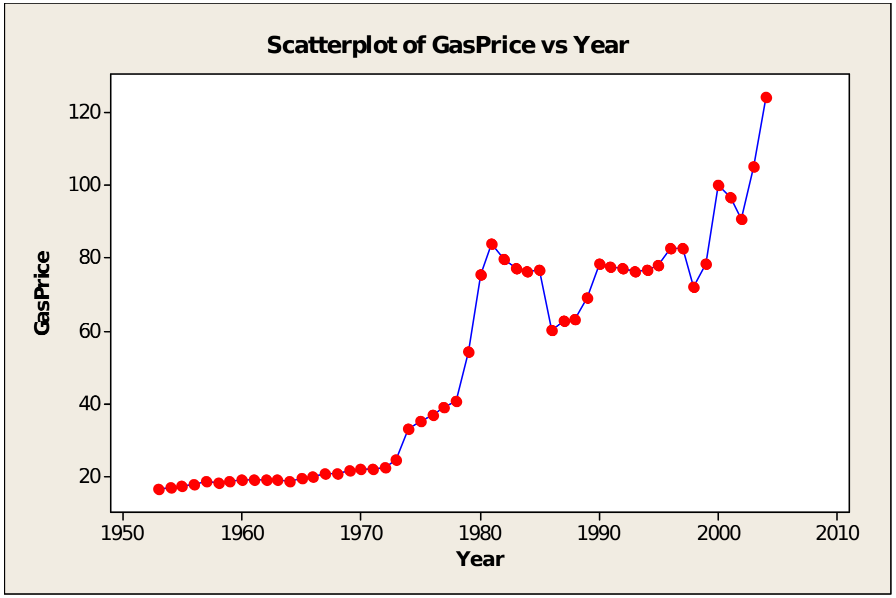
Does a rise in income cause a rise in gas prices ??????????????

US gasoline prices, 1953-2004, plotted against per-capita US income

A HIDDEN RELATIONSHIP

Not positively "related" to each other; both positively related to "time."





“RELATED” ...?

Want to capture: some variable X varies in the same direction and at the same scale as some other variable Y

$$cov(x,y) = \frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y})$$

What happens if:

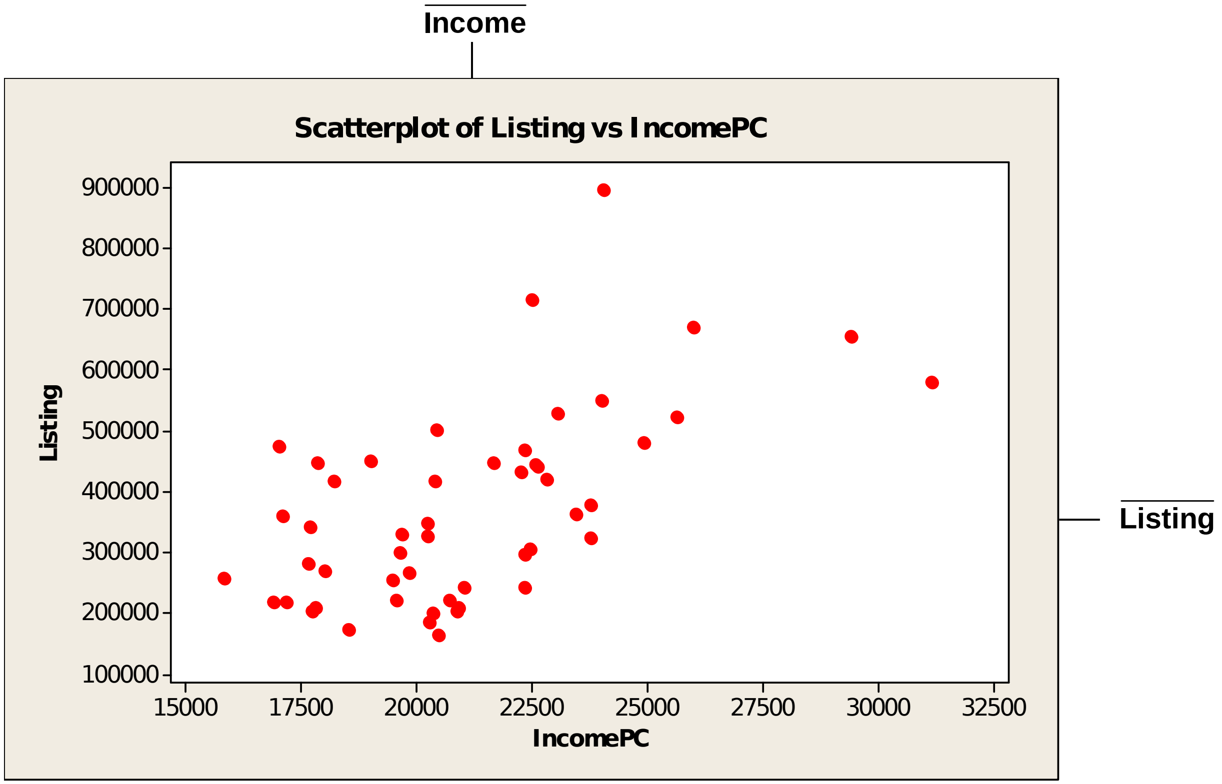
- X varies in the opposite direction as Y ????????
- X varies in the same direction as Y ????????

What are the units of the covariance ????????

Pearson’s correlation coefficient is unitless in [-1,+1]:

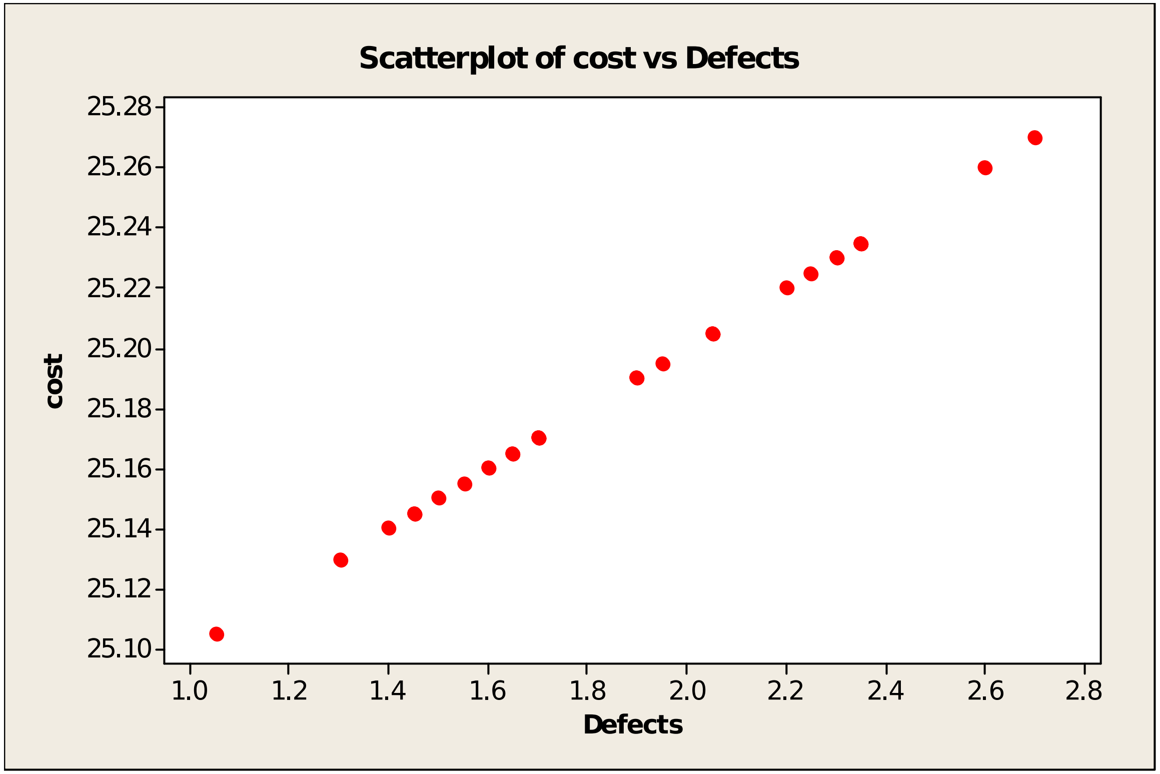
$$cor(x,y) = \frac{cov(x,y)}{sd(x)sd(y)}$$

CORRELATION

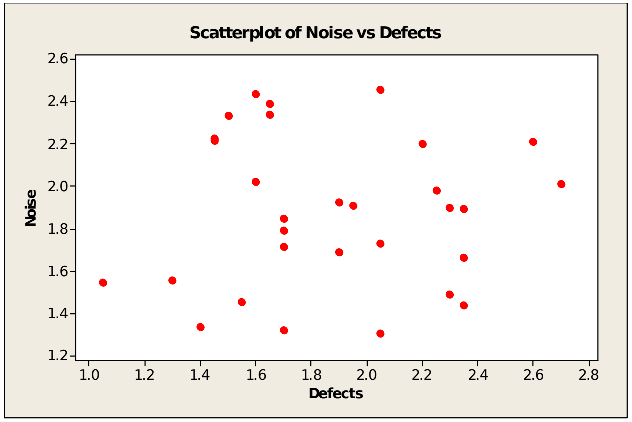


$$r_{\text{Income,Listing}} = +0.591$$

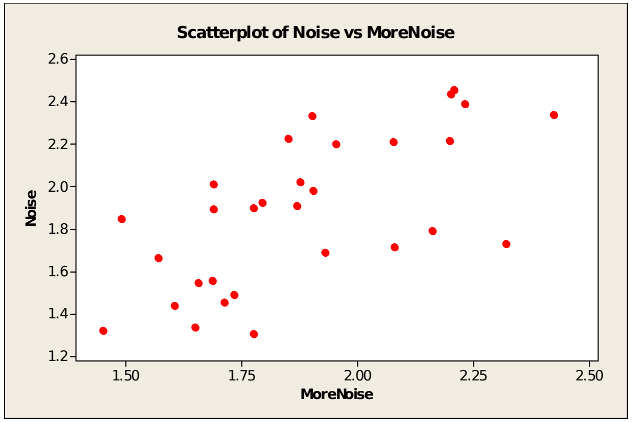
CORRELATIONS



r = +1.0

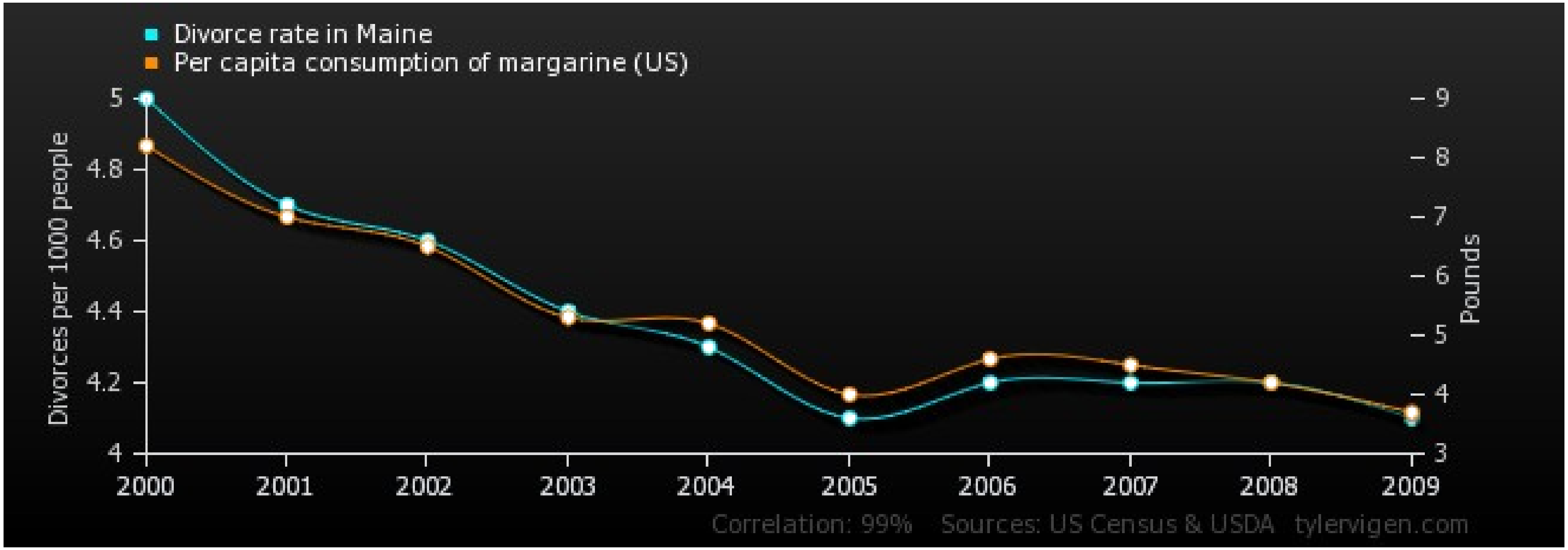


r = 0.0



r = +0.5

CORRELATION IS NOT CAUSATION!!!



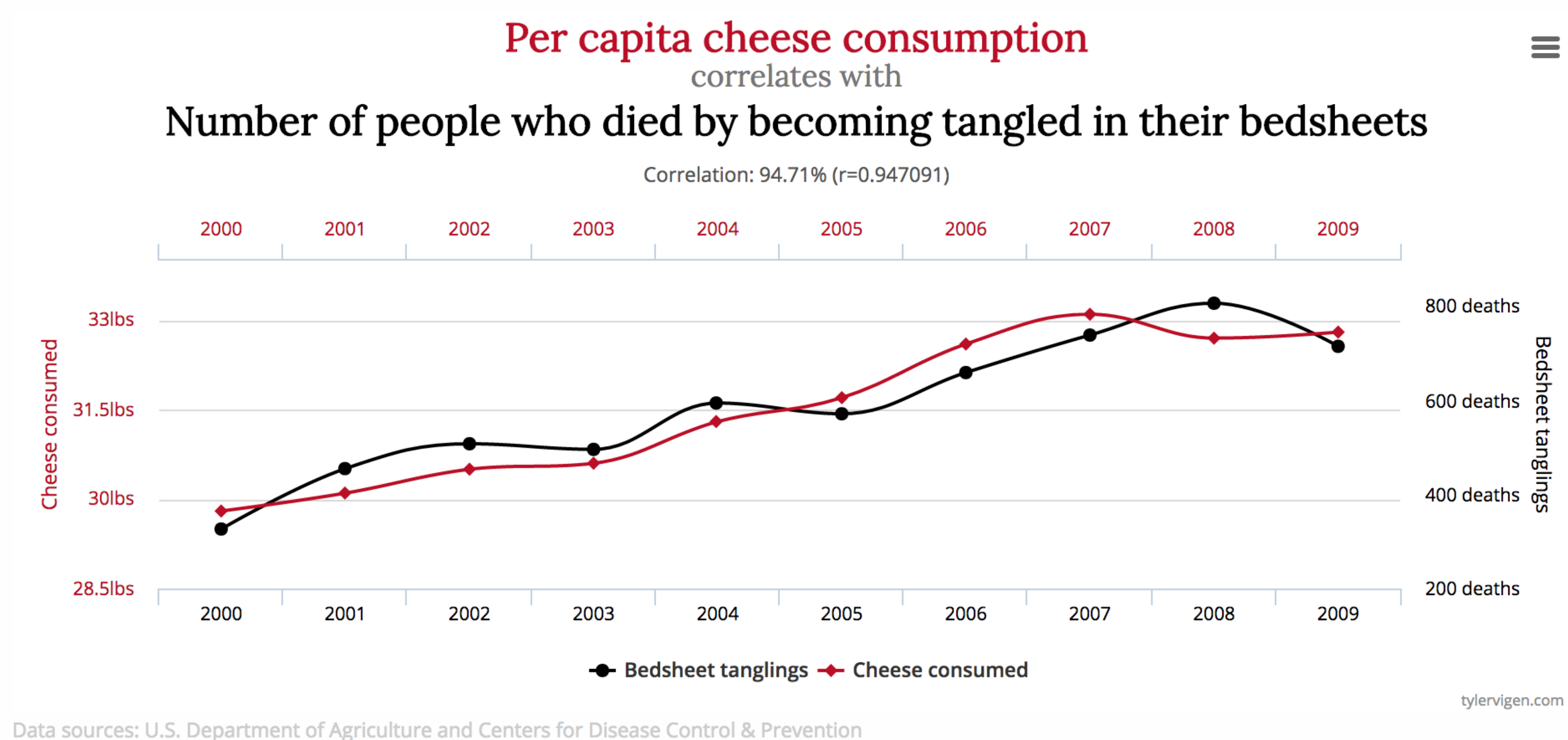
	2000	2001	2002	2003	2004	2005	2006	2007	2008	2009
Divorce rate in Maine Divorces per 1000 people (US Census)	5	4.7	4.6	4.4	4.3	4.1	4.2	4.2	4.2	4.1
Per capita consumption of margarine (US) Pounds (USDA)	8.2	7	6.5	5.3	5.2	4	4.6	4.5	4.2	3.7

r=0.993

?????????

http://tylervigen.com/spurious-correlations

JUST TO DRIVE THE POINT HOME …



TRANSFORMATIONS

TRANSFORMATIONS

So, you've figured out that your data are:

- Skewed
- Have vastly different ranges across datasets and/or different units

What do you do?

Transform the variables to:

- ease the validity and interpretation of data analyses
- change or ease the type of Stat/ML models you can use

STANDARDIZATION

Transforming the variable to a comparable metric

- known unit
- known mean
- known standard deviation
- known range

Three ways of standardizing:

- P-standardization (percentile scores)
- Z-standardization (z-scores)
- D-standardization (dichotomize a variable)

WHEN YOU SHOULD ALWAYS STANDARDIZE

When averaging multiple variables, e.g. when creating a socioeconomic status variable out of income and education.

When comparing the effects of variables with unequal units, e.g. does age or education have a larger effect on income?

Slides adapted from Maarten Buis

P-STANDARDIZATION

Every observation is assigned a number between 0 and 100, indicating the percentage of observation beneath it.

Can be read from the cumulative distribution

In case of knots: assign midpoints

The median, quartiles, quintiles, and deciles are special cases of P-scores.

	rent	cum %	percentile
room 1	175	5,3%	5,3%
room 2	180	10,5%	10,5%
room 3	185	15,8%	15,8%
room 4	190	21,1%	21,1%
room 5	200	26,3%	26,3%
room 6	**210**	31,6%	**36,8%**
room 7	**210**	36,8%	**36,8%**
room 8	**210**	42,1%	**36,8%**
room 9	230	47,4%	47,4%
room 10	**240**	52,6%	**55,3%**
room 11	**240**	57,9%	**55,3%**
room 12	**250**	63,2%	**65,8%**
room 13	**250**	68,4%	**65,8%**
room 14	280	73,7%	73,7%
room 15	**300**	78,9%	**81,6%**
room 16	**300**	84,2%	**81,6%**
room 17	310	89,5%	89,5%
room 18	325	94,7%	94,7%
room 19	620	100,0%	100,0%

P-STANDARDIZATION

Turns the variable into a ranking, i.e. it turns the variable into a ordinal variable.

It is a non-linear transformation: relative distances change

Results in a fixed mean, range, and standard deviation; M=50, SD=28.6, This can change slightly due to knots

A histogram of a P-standardized variable approximates a uniform distribution

Slides adapted from Maarten Buis

CENTERING AND SCALING

Transform your data into a unitless scale

- Put data into "standard deviations from the mean" units
- This is called standardizing a variable, into standard units

Given data points $x = x_1, x_2, ..., x_n$:

$$z_i = \frac{(x_i - \bar{x})}{\text{sd}(x)}$$

Translates x into a scaled and centered variable z

What is the mean of z ??????????

What is the standard deviation of z ??????????

CENTERING OR SCALING

Maybe you just want to center the data:

$$z_i = (x_i - \overline{x})$$

What is the mean of *z* ??????????

What is the standard deviation of z ??????????

Maybe you just want to scale the data:

$$z_i = \frac{x_i}{\mathrm{sd}(x_i)}$$

What is the mean of *z* ??????????

What is the standard deviation of z ??????????

DISCRETE TO CONTINUOUS VARIABLES

Some models only work on continuous numeric data

Convert a binary variable to a number ???????????

- health_insurance = {“yes”, “no”} {1, 0}

Why not {-1, +1} or {-10, +14}?

- 0/1 encoding lets us say things like “if a person has healthcare then their income increases by $X.”
- Might need {-1,+1} for certain ML algorithms (e.g., SVM)

DISCRETE TO CONTINUOUS VARIABLES

What about non-binary variables?

My main transportation is a {BMW, Bicycle, Hovercraft}

One option: { BMW ✉ 1, Bicycle ✉ 2, Hovercraft ✉ 3 }

- Problems ??????????

One-hot encoding: convert a categorical variable with N values into a N-bit vector:

- BMW ✉ [1, 0, 0]; Bicycle ✉ [0, 1, 0]; Hovercraft ✉ [0, 0, 1]

```
# Converts dtype=category to one-hot-encoded cols
cols = [‘my_transportation’]
df = df.get_dummies( columns = cols )
```

CONTINUOUS TO DISCRETE VARIABLES

Do doctors prescribe a certain medication to older kids more often? Is there a difference in wage based on age?

Pick a discrete set of bins, then put values into the bins

Equal-length bins:

- Bins have an equal-length range and skewed membership
- Good/Bad ????????

Equal-sized bins:

- Bins have variable-length ranges but equal membership
- Good/Bad ????????

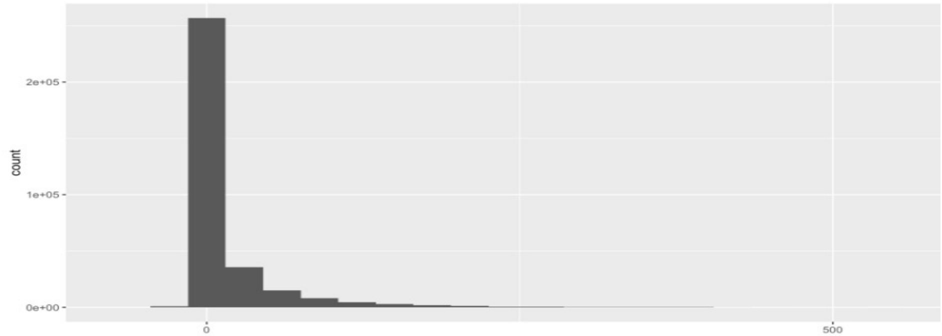
SKEWED DATA

Skewed data often arises in multiplicative processes:

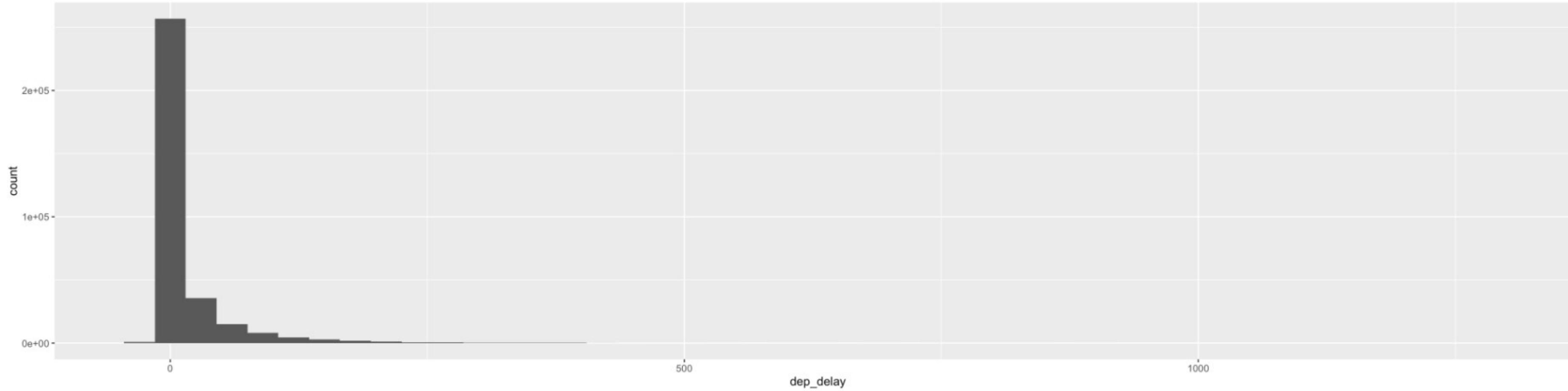
- Some points float around 1, but one unlucky draw ✉ 0

Logarithmic transforms reduce skew:

- If values are all positive, apply $\log_2$ transform
- If some values are negative:
  - Shift all values so they are positive, apply $\log_2$
  - Signed log: $\text{sign}(x) * \log_2(|x| + 1)$



SKEWED DATA



log$_2$ transform on airline takeoff delays